LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

SECOND SEMESTER – APRIL 2010

ST 2502/ST 2501/2500 - STATISTICAL MATHEMATICS - I

Date & Time: 20/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

PART – A

Answer ALL questions **(10 x 2 = 20 marks)**

1. Find $\lim_{x \to 3} f(x)$ where $f(x) = \frac{|x-3|}{x-3}, x \neq 3.$
2. Define mixed type distributions and give an example.
3. Define M.G.F. of a random variable.
4. Define absolute convergence of a series.
5. State Rolle's Theorem.
6. Define Taylor's expansion of a function about x = a.
7. Verify whether the vectors (1,0,0), (4,1,2) and (2,-1,4) are linearly independent.
8. If X and Y are two continuous random variables, define conditional distribution of X given Y = y.
9. Define an orthogonal matrix.
10. Define variance – covariance matrix of a bivariate distribution.

PART – B

Answer any FIVE questions **(5 x 8 = 40 marks)**

11. A function f(x) is defined as follows:

$$f(x) = \begin{cases} x^2 + x, & for\ x < 2 \\ 5 & for\ \ x = 2 \\ x^2 + 2x + 3 & for\ x > 2 \end{cases}$$

Verify whether the function f(x) is continuous at x = 1.

12. Let the random variable X have the distribution P(x=0) = P(x=2) = p ; P(x=1) = 1-2p for $0 \le p \le \frac{1}{2}$. For what value of p is the variance of X maximum?

13. Let X be a random variable with the probability generating function P(s). Find the P.G.F. of
(i) 3X (ii) X+5.

14. Give an example to show that a function which is contnuous at a point need not have derivative at that point.

15. Find a suitable point c of Rolle's theorem for f(x) = (x–3) (x-5) for $3 \le x \le 5$.

16. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 8 & 7 \\ 6 & 4 & 2 \end{bmatrix}$.

17. Let X be a continous random variable with the p.d.f. given by:

$$f(x) \begin{cases} ax, & 0 \le x < 1 \\ a, & 1 \le x < 2 \\ -ax + 3a, & 2 \le x < 3 \\ 0 & , otherwise \end{cases}$$

Determine the constant 'a' and find the distribution function of X.

(P.T.O.)

18. A random variable x has the following p.d.f.

$$f(x)\begin{cases} k(x-x^2), & 0\le x\le 1 \\ 0 & , otherwise \end{cases}.$$

Find the value of k.

Also find the mean and mode of the distribution.

PART – C

Answer any TWO questions **(2x20 = 40 marks)**

19. (i) Examine the continuity at x = 0,1,2 of the function f(x) defined below.

$$f(x)=\begin{cases} x^2 & x\le 0 \\ 2x & 0<x\le 1 \\ 4x^2-2x, & 1<x\le 2 \\ 10-x & x>2 \end{cases}.$$

(ii) Prove that a non-increasing sequence of real numbers which is bounded below is convergent.

20. (i) If the moments of a discrete random variable X are given by E $E[X^r]$ = 0.6 for r = 1,2,3,... showthat P(x=0)=0.4 ; P(x=1)=0.6 P(x=x) = 0, otherwise.

(ii) Find the M.G,F of the random variable X whose moments are $\mu_r^1 = r\,!$ for r = 1,2,3, ...

21. (i) If $f(x,y)=\dfrac{x+y}{x^2+y^2}$, find $\dfrac{\partial f}{\partial x},\dfrac{\partial f}{\partial y},\dfrac{\partial^2 f}{\partial x^2}$ *and* $\dfrac{\partial^2 f}{\partial y^2}$.

(ii) X and Y are jointly discrete random variables with the following joint probability distribution.

$$p(x,y)=\begin{cases} \dfrac{1}{4}, & x,y)=(-3,-5),(-1,1), \\ & (1,1),(3,5) \\ 0, & otherwise \end{cases}$$

Find the following (i) E(X) (ii) E(Y) (iii) E(XY) and (iv) $E(x\ \ y=1)$

22. (i) Express the matrix $A=\begin{bmatrix} 5 & 4 & 2 \\ 1 & 6 & 2 \\ 3 & 5 & 7 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

(ii) Find the rank of the matrix $A=\begin{bmatrix} 1 & 5 & 6 \\ 0 & 0 & 0 \\ 2 & 4 & 5 \end{bmatrix}$.

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